Quadcopter modeling and control

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1 Introduction

Quadcopters have recently been used in many application domains, like surveillance, security or even in transport. One of their usual tasks is to track a trajectory. For example to avoid certain obstacles the drone should follow a well defined path. However tracking any kind of trajectory is a difficult control problem, in many cases a simple linear controller will provide acceptable results. In our project we investigate wether advanced control design technologies can ensure better performances. This paper compares a linear and a fuzzy controller.

Our mathematical model for the UAV is based on the AR Drone 2 quadcopter. Using the Euler-Lagrange approach the dynamic model of the quadcopter can be obtained. Based on (Dydek *et al.*, 2013), the dynamics have the form in (1). In the model, x, y and z represent the position of the drone, while ϕ, θ and ψ represent the Euler angles ; the first derivative is used for the linear and angular velocities, while the second derivative for the linear and angular accelerations. We have the state vector $X = \begin{bmatrix} \zeta & \dot{\zeta} & \eta & \dot{\eta} \end{bmatrix}$, where $\zeta = \begin{bmatrix} x & y & z \end{bmatrix}$ and $\eta = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T$ and the input vector $U = \begin{bmatrix} U_{coll} & U_\phi & U_\theta & U_\psi \end{bmatrix}^T$.

Based on the model in (1), our goal is to track a certain trajectory. The main focus for this tracking was the latitude and the longitude, while for the altitude a fixed reference was used. In order to see the capability of the controllers the trajectory used is the one in Fig. 1. Thus the main task is to follow this trajectory and at the same time keep the altitude at a certain level.

2 Linear quadratic tracking

The first option which can be used is a linear quadratic tracking control. We first linearize the system around an equilibrium point, where the drone is hovering ; the controller has the form :

$$u(t) = -KX(t)$$

where K is the controller gain computed by minimizing a quadratic performance function. Because we want to achieve a tracking control, the derivative of the tracking errors are introduced as auxiliary states. Using the linear controller on the nonlinear system (1) a good tracking was achieved for x, y and z, as can be seen in Fig. 1.

3 Takagi-Sugeno (TS) fuzzy control

The dynamics of the UAV described in (1) are highly nonlinear. To actually take into account the nonlinearities we use a nonlinear TS controller (Takagi & Sugeno, 1985). For this, we assume that ϕ, θ, and ψ are varying on the range $\left[-\frac{\pi}{6}, \quad \frac{\pi}{6}\right]$. We have the following approximations :

$$\begin{aligned} \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) &\approx 0.9\theta \\ \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) &\approx -0.95\phi, \\ \cos(\phi)\cos(\theta) &\approx 0.9 \end{aligned}$$

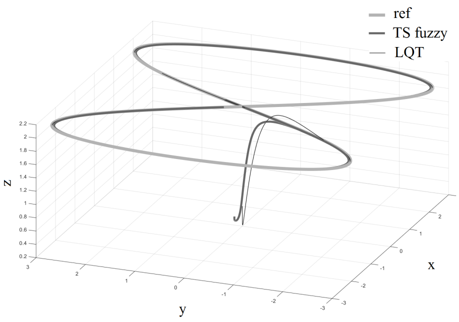

FIGURE 1 – Tracking with TS fuzzy control and LQR

$$\begin{aligned}
\ddot{x} =& [\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)]\frac{U_{coll}}{m} \\
\ddot{y} =& [\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi))\frac{U_{coll}}{m} \\
\ddot{z} =& -g + \cos(\phi)\cos(\theta)\frac{U_{coll}}{m} \\
\ddot{\phi} =& \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + \frac{1}{I_x}U_\phi \\
\ddot{\theta} =& \dot{\phi}\dot{\psi}\frac{I_z - I_x}{I_y} + \frac{1}{I_y}U_\theta \\
\ddot{\psi} =& \dot{\phi}\dot{\theta}\frac{I_x - I_y}{I_z} + \frac{1}{I_z}U_\psi
\end{aligned} \tag{1}$$

Even with these approximations the system remains highly nonlinear. Using the sector nonlinearity approach (Ohtake *et al.*, 2001), the approximate system can be converted into a TS fuzzy model. The controller used has the form :

$$u(t) = -K(X(t))X(t)$$

It is a nonlinear control, because K depends on $X(t)$. Based on this, a tracking controller (Lendek *et al.*, 2011) has been computed, which achieved the results in Fig. 1.

4 Conclusion and future work

As it can be seen both options are working, for our trajectory. However the linear one has a strange behaviour in the transient regime, which can be partially seen in Fig. 1. That can be due to the fact that the system is highly nonlinear. Furthermore, no theoretical guarantees exist on the region where the linear controller is working. In our future work we will include in the control also the angle with respect to the z axis, i.e. we want to point in the direction of the trajectory with the UAV.

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